Trigonometry Identities: How We Teach It

Sample summary note for reference only

Worked examples (Examples 1–3)

The tutor walks through each step and explains the reason for every line. Students focus on understanding the thinking, not just copying the answer.

Blank examples (Examples 4–5)

Students get a few minutes to attempt the question first, depending on how hard it is. Then we go through the walkthrough together and ask: **Did I get this right? If not, why not? What was I thinking?**

Finding where the thinking went off track is how students improve.

This note is a summary of what the tutor explains in class. The reasoning is taught live, step by step, and students ask questions as we go.

The examples

Example 1 (Prove an identity)

Worked example: tutor explains every step

Prove that $\sec x - \cos x \equiv \sin x \tan x$.

Count the terms. Left side: 2 terms. Right side: 1 term. So we combine the left side first.

$$\text{LHS} = \frac{1}{\cos x} - \cos x$$

sec x = 1 ÷ cos x

$$= \frac{1 - \cos^2 x}{\cos x}$$

2 terms on the left, 1 on the right: combine into 1 term (common denominator).

$$= \frac{\sin^2 x}{\cos x}$$

1 − cos²x = sin²x

$$= \sin x \times \frac{\sin x}{\cos x}$$

$$= \sin x \tan x$$

= RHS (shown)

Example 2 (Prove, then "hence") Worked example: tutor explains every step

(a) Prove the identity $\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x}{1+\cos x} \equiv \dfrac{2}{\sin x}$.

(b) Hence, find all the angles between 0° and 360° for which $\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x}{1+\cos x} = 3$.

(a) Count the terms. Left side: 2 terms. Right side: 1 term. Combine the left side.

$$\text{LHS} = \frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$$

$$= \frac{(1+\cos x)^2 + \sin^2 x}{\sin x(1+\cos x)}$$

2 terms on the left, 1 on the right: combine into 1 term (common denominator).

$$= \frac{1 + 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1+\cos x)}$$

expand $(1+\cos x)^2$

$$= \frac{2 + 2\cos x}{\sin x(1+\cos x)}$$

$\sin^2 x + \cos^2 x = 1$

$$= \frac{2(1+\cos x)}{\sin x(1+\cos x)}$$

factorise the top

$$= \frac{2}{\sin x}$$

= RHS (shown)

(b) By (a), the equation becomes $\dfrac{2}{\sin x} = 3$.

$$\sin x = \frac{2}{3}$$

Basic angle $\alpha = 41.8°$. $\sin x$ is positive: 1st and 2nd quadrants.

ANSWER $x = 41.8°, 138.2°$

Example 3 (Solve in radians) Worked example: tutor explains every step

Find all the angles between 0 and π inclusive which satisfy the equation $2\cos^2 x + 3\sin x = 3$.

We need one trig function only. There is a $\cos^2 x$, so use the Pythagorean identity.

$$2\cos^2 x + 3\sin x = 3$$

$$2(1 - \sin^2 x) + 3\sin x = 3$$

$\cos^2 x = 1 - \sin^2 x$

$$2\sin^2 x - 3\sin x + 1 = 0$$

collect on one side: a quadratic in sin x

$$(2\sin x - 1)(\sin x - 1) = 0$$

factorise

$\sin x = \dfrac{1}{2}$ gives $x = \dfrac{\pi}{6}$ or $\pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$. $\sin x = 1$ gives $x = \dfrac{\pi}{2}$.

ANSWER $x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$

Example 4 (Changed angle) Blank example: attempt first, tutor then walks through

Find all the angles between 0° and 360° which satisfy each of the following equations.

(a) $\cos(x + 30°) = -0.3$

(b) $\sin 2x = 0.866$

YOUR ATTEMPT

Check yourself after the walkthrough

- Did I get this right?
- If not, why not?
- What was I thinking at that step?

Example 5 (Exact angles) Blank example: attempt first, tutor then walks through

(a) Prove the identity $(\cos x + \sin x)(1 - \cos x \sin x) \equiv \cos^3 x + \sin^3 x$.

(b) Hence, find all the angles between $-\pi$ and π which satisfy the equation $(\cos x + \sin x)(1 - \cos x \sin x) = 0$, giving your answers in an exact form.

YOUR ATTEMPT

Check yourself after the walkthrough

- Did I get this right?
- If not, why not?
- What was I thinking at that step?